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1989 J. Phys. A: Math. Gen. 22 3113

(http://iopscience.iop.org/0305-4470/22/15/025)

J. Phys. A: Math. Gen. 22 (1989) 3113-3122. Printed in the UK

Kinetic growth walks and trails on oriented square lattices: hull percolation and percolation hulls

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Received 1 September 1988

Abstract. We have studied kinetic growth walks on the Manhattan lattice and kinetic growth trails on the L lattice. We find that the directional properties of these lattices prevent the kinetic growth walks from being trapped (except at the origin). As a consequence, KGWs are found to be in the SKW universality class for these lattices. Extensive numerical calculations support this fact. Moreover it has been shown that the recently introduced hull percolation is equivalent to the kinetic growth walks on the Manhattan lattice, and the bond percolation hull on the square lattice at the percolation threshold is equivalent to the kinetic growth trails on the L lattice. Our numerical estimates agree nicely with the original estimates, and thus support the equivalence.

1. Introduction

The 'hull' of a percolation cluster is defined as the set of vacant nearest-neighbouring bonds of the cluster that can be joined by a continuous path from infinity without crossing the cluster. Mandelbrot (1982) introduced this term to describe the external perimeter of a cluster. The hull can be considered as the surface of a cluster and thus it may be related to the surface energy.

Several attempts have been made to estimate the fractal dimension D_h of the hull at the percolation threshold. It has been conjectured that D_h is related to the percolation correlation length exponent ν by $D_h = 1 + 1/\nu$ (Sapoval *et al* 1985) (in two dimensions this gives $D_h = \frac{7}{4}$). Ziff *et al* (1984) introduced a random walk algorithm to generate the hull of the site percolation cluster (without generating the actual percolation cluster). Gunn and Ortuno (1985) devised another walk algorithm which represents the hull of the bond percolation cluster on the square lattice at the percolation threshold. Numerical studies (Voss 1984, Weinrib and Trugman 1985, Ziff 1986, Grassberger 1986) confirmed this conjecture in the two-dimensional case. Finally Saleur and Duplantier (1987) obtained the exact value of the hull dimension $D_h = \frac{7}{4}$ by mapping the two-dimensional percolation problem into a solvable Coulomb gas problem.

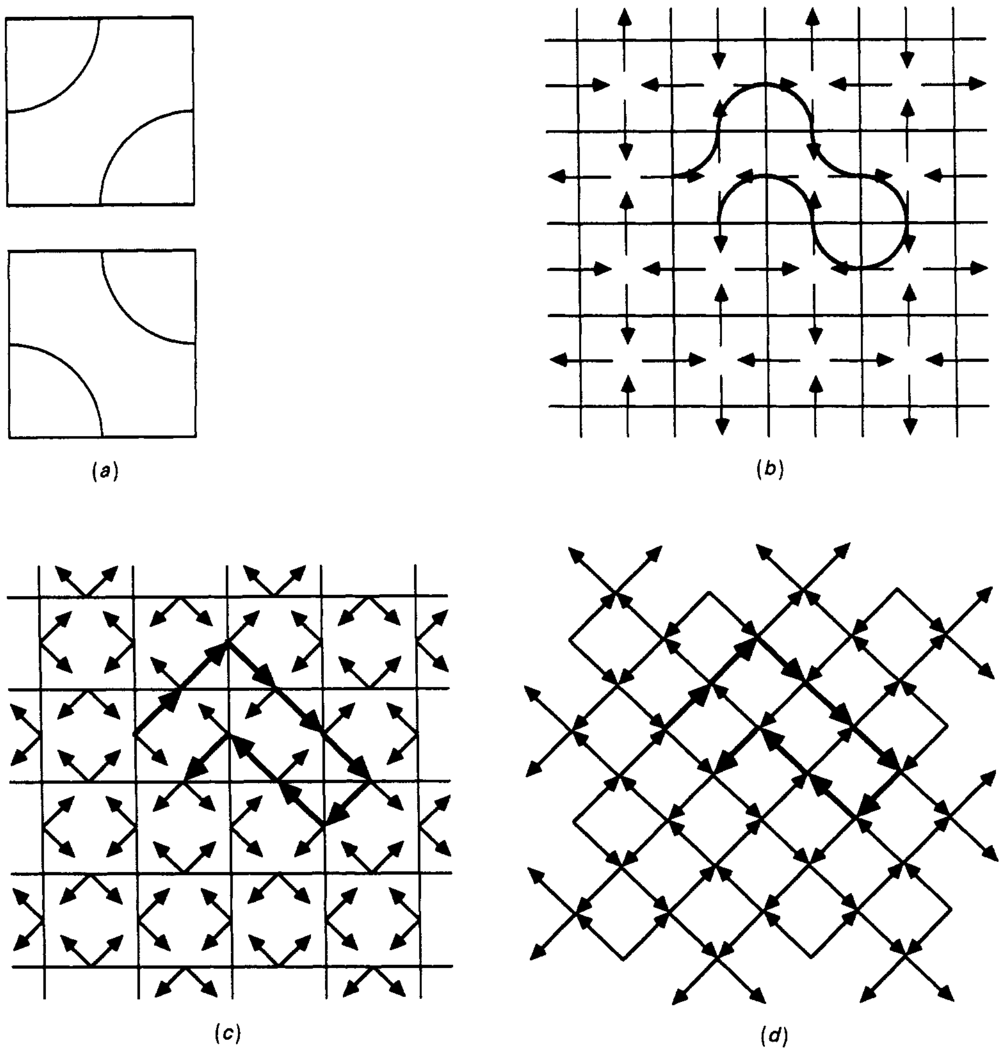
Self-avoiding walks (SAW) (de Gennes 1979) form a subset of random walks in which no walk has visited a site more than once. These walks carry equal weights, as they are random walks. They are used as a model of linear polymers in dilute solution with excluded volume interaction. However, to represent a growing polymer the weight of a walk configuration should depend on the configuration itself. This is because at every stage of growth, the number of options determines the weight. Kinetic growth walks (KGW) (Majid *et al* 1984, Hemmer and Hemmer 1984, Lyklema and Kremer 1984) are an example of such growing walks. They consist of the same set of walks as SAWs but each has a weight depending on its growth process. Specifically in this walk, the walker first determines the unvisited nearest-neighbour sites and then selects one of them randomly. The weight of each step is 1/(number of allowed sites), while the weight of a walk is the product of the individual step weights. KGWs belong to the same universality class as ordinary SAWs (Peliti 1984, Pietronero 1985, Lyklema and Kremer 1986). Weinrib and Trugman (1985) introduced another growing walk and called it the smart kinetic walk (SKW). Unlike SAWs and KGWs, this walk is never trapped. This makes SKWs create a new universality class, distinct from SAWs. In this case the walker first determines the unvisited neighbouring sites. From these sites only those which will never lead to any trapping situation are permitted. One of these sites is then chosen randomly for the next step. SKW rings in which the initial and final sites are the same are shown to corresponding precisely to the hull of the site percolation clusters on the triangular lattice at the percolation threshold on the dual honeycomb lattice (Weinrib and Trugman 1985). SKWs are also known by the alternative name indefinitely growing self-avoiding walks (IGSAW) introduced by Kremer and Lyklema (1985).

In this paper we consider KGWs on the Manhattan lattice and kinetic growth trails (KGTs) on the L lattice. We see that because of the directional properties of these lattices, KGWs and KGTs on these lattices behave similarly to SKWs, i.e. they are never trapped. The 'smartness' is thus a property of the lattice. In § 2 we show that kinetic growth walks on the Manhattan lattice are equivalent to the recently introduced hull percolation problem. In § 3 we show that kinetic growth trails on the L lattice are equivalent to the bond percolation hulls on the square lattice at the percolation threshold. Section 4 contains our conclusion.

2. Kinetic growth walks on the Manhattan lattice and hull percolation

Recently Roux *et al* (1988) introduced a problem of random tiling in two dimensions that generates lines which are hulls of a 'non-standard' percolation model and called this problem 'hull percolation'. It was shown that these lines obey the same statistics as hulls of ordinary percolation clusters. In that model they used tiles of two different types as shown in figure 1(a). These are squares which have at the opposite corners quarters of a circle with radius equal to half of the length of the square and centred on opposite vertices. On one square the circles occupy left-top and right-bottom corners and on the other square they are placed at right-top and left-bottom corners. Now on a large square lattice every small square is filled with one of these two tiles with equal probability. By construction, the circles on the tiles at adjacent squares (squares having an edge common) form continuous lines. As a result, the whole lattice is filled up with closed lines of different lengths and open lines which both start and end at infinity.

One such single line can be generated by deciding the orientation of the squares with equal probability only along the line being generated. When an already visited square is reached the line is continued through the remaining unvisited arc of the square and for this step no new decision needs to be taken as it is pre-ordained. Therefore the generating line has two possible options when a new square is reached, and only one option to continue when a previously visited square is encountered. Once one arc is traversed in a certain direction, the other arc will be traversed in the opposite direction, by construction. As a result, when such a path starts, its starting direction determines the directions in which all other lattice edges will be traversed.



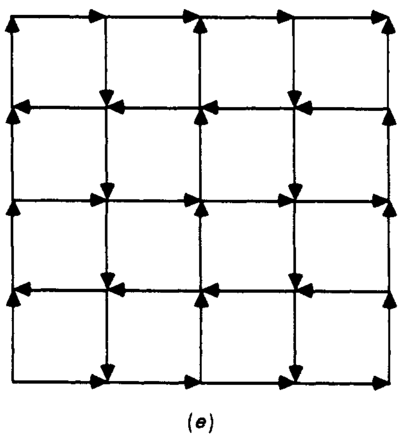


Figure 1. (*a*) Two tiles with circles in opposite corners. (*b*) Generation of a single line by the hull percolation process. Once it is started, the directions (inward or outward) in which all other bonds will be crossed is determined. (*c*) The possible stepping directions at each bond are shown. (*d*) The underlying square lattice is deleted and the possible options are retained. The directions are the same as those on the Manhattan lattice. (*e*) The Manhattan lattice: alternate rows (or columns) parallel, adjacent rows (or columns) antiparallel.

Two opposite sides of a square are crossed in the outward direction while the other two sides are crossed in the inward direction (see figure 1(*b*)). Each of these sides has two possible directions of crossing (see figure 1(*c*)). Now if we consider only these directions and forget the underlying lattice what we get is figure 1(*d*). From this figure we see that the possible options are arranged exactly as allowed paths on a Manhattan lattice.

On a Manhattan lattice alternate rows (or columns) are parallel, and adjacent rows (or columns) are antiparallel (see figure 1(*e*)). Self-avoiding walks on the Manhattan lattice were first studied by Malakis (1975). Numerical studies on this problem suggested that these walks belong to the same universality class as that of the ordinary SAWs (Malakis 1975, Enting and Guttmann 1985). When a self-avoiding walker on this lattice visits a neighbouring site to its previously visited path a cage is formed. On the square lattice the walker has to be 'smart' enough to find a path which will prevent it entering the cage. On the Manhattan lattice this is automatic. As two adjacent rows (or columns) are antiparallel, the walker has only one option for the next step which directs the walk outward. Because of this property, a KGW will not normally be trapped. The only situation in which this walker can be trapped is at a neighbouring site of the starting position, in which case it forms a loop. As a result, KGWs on the Manhattan lattice are also 'smart', and it is the directional property of the lattice which is responsible for this behaviour. We expect that unlike KGWs on regular lattices KGWs on the Manhattan lattice should behave in the same way as SKWs.

Therefore we see that for hull percolation and KGWs on the Manhattan lattice, the generating line and the walker have two options in general, except when they fall on a site adjacent to a previously visited site, when there is only one option. They are never trapped except at the origin. From these similarities, we conclude that hull percolation and KGWs on the Manhattan lattice are the same.

To establish this conclusion on a stronger basis we have studied this walk extensively by numerical methods. We enumerated exactly ordinary SAWs and KGWs on the Manhattan lattice up to 37 steps (see table 1). For ordinary SAWs this is nine terms further than the series data available in the literature (Malakis 1975). We analysed the series by the method of differential approximants, constructing first order inhomogeneous differential approximants precisely as described in Guttmann (1987). To save space we do not give our dozens of tables of data, but just report the conclusions.

For the SAW generating function, we observe that, as the number of terms increases, so do the estimates of x_c and γ. Using most or all terms gives estimates $x_c = 0.576\,82$ and $\gamma \geqslant 1.330$. Assuming that the increasing estimates of γ have as their limit $\gamma = \frac{43}{32} = 1.343\,75$, which is the γ value for SAWs on regular lattices, we find $x_c \simeq 0.576\,93$. This is in agreement with the earlier estimate of Enting and Guttmann (1985). The sum of the squared end-to-end distance series gives $x_c \simeq 0.576\,78$, $\gamma + 2\nu = 2.812$. Again, there is an increasing trend of x_c and γ values with increasing numbers of terms. With $x_c = 0.576\,93$, the (biased) estimate of $\gamma + 2\nu$ increases to 2.840. With $\gamma = \frac{43}{32}$, this yields $\nu = 0.748$. The mean square end-to-end distances $\langle R^2_{NW} \rangle$ for KGWs weighted by the individual configuration weights were analysed similarly, using the fact that the 'critical point' of the generating function $\Sigma \langle R^2_{NW} \rangle x^n$ is at 1. In this way, we obtained an exponent of 2.152, hence $\nu_{KGW} = 0.576$. All exponent estimates are uncertain in the last quoted digit. The fourth column of table 1 gives the series data for the sum of the weights of all N-step KGW configurations, which we denote w_N.

We have also studied KGWs by Monte Carlo methods in two different ways. We simulated one million configurations for each of walk lengths $N = 16$, 20, 24, 32, 40,

Table 1. Series data for the number of distinct configurations (C_N), sum of the squares of end-to-end distances (R_N^2) for SAWS, the mean square end-to-end distances ($\langle R_{NW}^2 \rangle$), and the total weight (W_N) for KGWs on the Manhattan lattice.

N	C_N	R_N^2	$\langle R_{NW}^2\rangle$	W_N
1	1	1	1.000 000 0000 E+00	5.000 000 0000 E−01
2	2	6	3.000 000 0000 E+00	5.000 000 0000 E−01
3	4	20	5.000 000 0000 E+00	5.000 000 0000 E−01
4	7	56	7.250 000 0000 E+00	5.000 000 0000 E−01
5	13	141	9.500 000 0000 E+00	5.000 000 0000 E−01
6	24	328	1.150 000 0000 E+01	5.000 000 0000 E−01
7	44	732	1.356 250 0000 E+01	5.000 000 0000 E−01
8	77	1 584	1.624 193 5483 E+01	4.843 750 0000 E−01
9	139	3 339	1.845 161 2903 E+01	4.843 750 0000 E−01
10	250	6 894	2.062 500 0000 E+01	4.843 750 0000 E−01
11	450	14 018	2.282 661 2903 E+01	4.843 750 0000 E−01
12	788	28 132	2.567 252 0661 E+01	4.726 562 5000 E−01
13	1 403	55 819	2.797 107 4380 E+01	4.726 562 5000 E−01
14	2 498	109 668	3.024 328 5123 E+01	4.726 562 5000 E−01
15	4 447	213 711	3.250 258 2644 E+01	4.726 562 5000 E−01
16	7 782	413 520	3.561 264 5348 E+01	4.619 140 6250 E−01
17	13 769	795 041	3.798 110 4651 E+01	4.619 140 6250 E−01
18	24 363	1 519 632	4.031 355 7082 E+01	4.619 140 6250 E−01
19	43 106	2 890 050	4.264 762 8171 E+01	4.619 140 6250 E−01
20	75 396	5 471 864	4.589 130 7276 E+01	4.530 563 3544 E−01
21	132 865	10 317 249	4.830 715 0194 E+01	4.530 563 3544 E−01
22	234 171	19 378 344	5.070 154 5475 E+01	4.530 563 3544 E−01
23	412 731	36 273 795	5.309 476 3021 E+01	4.530 563 3544 E−01
24	721 433	67 690 580	5.648 466 8808 E+01	4.452 466 9647 E−01
25	1 267 901	125 950 933	5.895 060 9906 E+01	4.452 466 9647 E−01
26	2 228 666	233 715 516	6.139 817 2690 E+01	4.452 466 9647 E−01
27	3 917 654	432 618 758	6.384 399 8795 E+01	4.452 466 9647 E−01
28	6 843 596	798 985 468	6.733 037 5464 E+01	4.384 962 3203 E−01
29	12 004 150	1 472 443 438	6.984 032 7877 E+01	4.384 962 3203 E−01
30	21 059 478	2 708 024 388	7.233 411 4264 E+01	4.384 962 3203 E−01
31	36 947 904	4 971 153 360	7.482 624 3431 E+01	4.384 962 3203 E−01
32	64 506 130	9 109 787 516	7.841 342 5632 E+01	4.324 709 6426 E−01
33	112 983 428	16 666 330 020	8.096 351 0414 E+01	4.324 790 6426 E−01
34	197 921 386	30 442 830 024	8.349 961 4208 E+01	4.324 709 6426 E−01
35	346 735 329	55 525 884 185	8.603 399 2446 E+01	4.324 709 6426 E−01
36	605 046 571	101 137 051 948	8.970 239 3834 E+01	4.270 777 1644 E−01
37	1058 544 744	183 972 829 656	9.228 939 2120 E+01	4.270 777 1644 E−01

48, ..., 320, 384, 512. We fit the mean square end-to-end distances $\langle R_N^2\rangle$ to a form $N^{2\nu_{\mathrm{KGW}}}$. A log-log plot of $\langle R_N^2\rangle$ against N gives the value of $\nu_{\mathrm{KGW}} = 0.573$ (see figure 2). In the second method we study kinetic growth walk loops. On an $L\times L$ lattice (L even) the walker starts from a site near the centre and walking is continued until it revisits the origin. With toroidal boundary conditions on this lattice we considered those walk configurations which have visited the four boundaries at least once. The lattice sizes were $L = 16$, 20, 24, 32, 40, 48, ..., 160, 192, 256, and the number of configurations at each size was varied from 50 000 to 100 000. We fitted the average loop length $\langle P_L\rangle$ to a form $\langle P_L\rangle \sim L^{D_{\mathrm{KGW}}}$ where D_{KGW} denotes the fractal dimension of the KGWS. A log-log plot of $\langle P_L\rangle$ against L gives a value of $D_{\mathrm{KGW}} = 1.74$ (see figure 2).

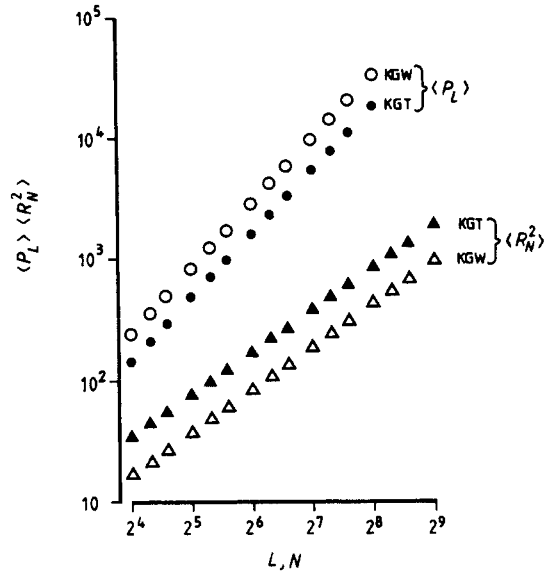


Figure 2. On a log-log plot, points denoted by (○) correspond to $\langle P_L \rangle$ for KGW on the Manhattan lattice, points denoted (●) correspond to $\langle P_L \rangle$ against L for KGT on the L lattice, points denoted (△) correspond to $\langle R_N^2 \rangle$ against N for KGW on the Manhattan lattice, while points denoted (▲) correspond to $\langle R_N^2 \rangle$ against N for KGT on the L lattice.

We have also studied indefinitely growing self-avoiding walks on the Manhattan lattice up to a walk length $N = 36$. The number of distinct configurations C_N and the mean square end-to-end distances $\langle R_{NW}^2 \rangle$ are given in table 2. We have analysed these in the same manner as we analysed the Manhattan lattice data. Here too we found a steady, increasing trend of estimates of x_c and γ with the number of coefficients used. The last entries gave $x_c = 0.57\,684$ and $\gamma = 1.333$. Assuming $\gamma = \frac{43}{32}$ as above gave $x_c = 0.576\,92$. Analysis of the generating function $\Sigma \langle R_{NW}^2 \rangle x^n$ biased at $x_c = 1$ gave $\nu = 0.566$. To confirm that the exponents $\nu_{\mathrm{KGW}}(\text{Manhattan}) = \nu_{\mathrm{IGSAW}}(\text{Manhattan})$ we studied the ratio $r_n = \langle R_{NW}^2 \rangle_{\mathrm{KGW}} / \langle R_{NW}^2 \rangle_{\mathrm{IGSAW}}$ using the method of Guttmann (1985). Assuming $r_n \sim N^x$, we first form the sequence $\phi_N = \ln(r_N / r_{N-4}) / \ln(N/N-4))$ which gives estimates of the exponent x, and accounts for the four-term periodicity of terms associated with the Manhattan lattice. Linear extrapolants of the squence ϕ_N are given by the sequence θ_N, defined by $\theta_N = [N\phi_N - (N-4)\phi_{N-4}]/4$. From these sequences we estimate $|x|$ to be less than 0.004. This shows convincingly that KGWs on the Manhattan lattice belong to the SKW universality class.

3. Kinetic growth trails on the L lattice and percolation hull

Gunn and Ortuno (1985) studied a model of deterministic classical dynamics in a random environment and related it to a generalised form of percolation. The model is defined as follows. On a square lattice, a rotation matrix $R(\theta)$ is associated with each site. The angle of rotation θ may take four different values namely 0, $\pi/2$, π and $-\pi/2$. The angular variables at different sites are assigned randomly from a probability distribution $p(\theta)$. Random walks are considered on such a lattice. The

Table 2. Series data for the number of distinct configurations (C_N) and the mean square end-to-end distances ($\langle R^2_{NW}\rangle$) for IGSAWS on the Manhattan lattice.

N	C_N	$\langle R^2_{NW}\rangle$
1	1	1.000 000 000 00 E+00
2	2	3.000 000 000 00 E+00
3	4	5.000 000 000 00 E+00
4	7	7.250 000 000 00 E+00
5	13	9.500 000 000 00 E+00
6	24	1.150 000 000 00 E+01
7	43	1.368 750 000 00 E+01
8	77	1.601 562 500 00 E+01
9	139	1.821 875 000 00 E+01
10	249	2.042 578 125 00 E+01
11	443	2.275 000 000 00 E+01
12	786	2.510 351 562 50 E+01
13	1 400	2.744 335 937 50 E+01
14	2 486	2.977 758 789 06 E+01
15	4 395	3.215 136 718 75 E+01
16	7 758	3.458142 089 84 E+01
17	13 732	3.700 244 140 62 E+01
18	24 251	3.940 536 499 02 E+01
19	42 710	4.185 334 777 83 E+01
20	75 154	4.434 034 729 00 E+01
21	132 487	4.681 660 461 42 E+01
22	233 173	4.928 848 457 33 E+01
23	409 617	5.179 501 152 03 E+01
24	719 157	5.433 085 918 42 E+01
25	1 264 303	5.686 174 154 28 3+01
26	2 219 916	5.939 038 574 69 E+01
27	3 892 603	6.194 674 378 63 E+01
28	6 822 808	6.452 844 782 17 E+01
29	11 970 965	6.710 737 723 11 E+01
30	20 983 162	6.968 471 668 28 E+01
31	36 742 831	7.228 612 907 23 E+01
32	64 319 064	7.490 943 145 00 E+01
33	112 682 333	7.753 087 267 46 E+01
34	197 256 386	8.015 208 492 94 E+01
35	345 033 765	8.279 411 694 12 E+01
36	603 376 033	8.545 521 281 08 E+01

walker starts from an arbitrarily chosen site of the lattice with an arbitrarily chosen direction. At every site visited by the random walker the rotation matrix at that site rotates the incoming direction into the outgoing direction. These trajectories are either infinite (i.e. starting and ending at infinity) or cyclic.

Consider the particular case of the probability distribution $P(\pi/2) = P(-\pi/2) = \frac{1}{2}$ and $P(0) = P(\pi) = 0$. The paths generated by this distribution are the perimeters of bond percolation clusters on the square lattice at the percolation threshold. The argument for this result is outlined by Gunn and Ortuno (1985). Paths generated by the above probability distribution must turn left or right at each step. Any site on such a path may be revisited only by a step in the same direction (but opposite sense) as the previously visiting step. If barriers are drawn diagonally across the sites visited by the trajectory, then the cluster of barriers enclosed by the trajectories represent the

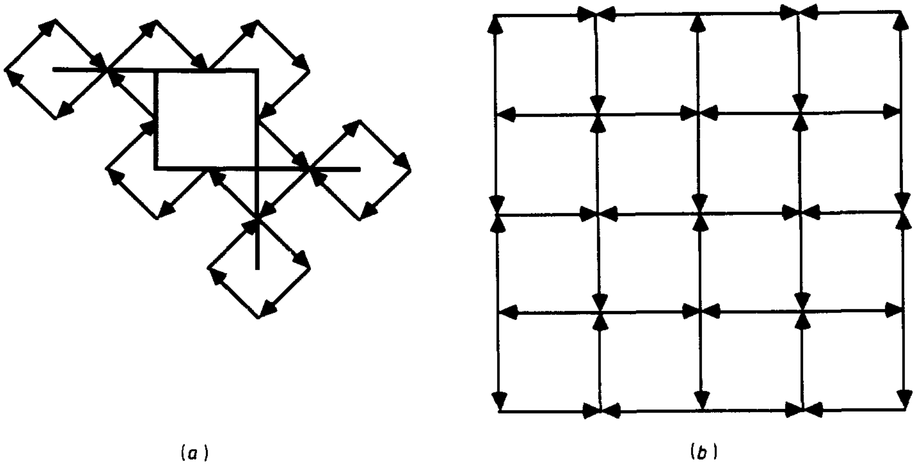


Figure 3. (*a*) The path of a random walk generated by the algorithm of Gunn and Ortuno (thin line with arrows on it) which is the perimeter of the bond percolation cluster (bold line). (*b*) The L lattice: every bond on a path is at right angles to its predecessor.

percolation clusters at the percolation threshold, and the trajectory is its hull (see figure 3(*a*)). A single such trajectory can be generated on a pure square lattice by a random walker which turns either left or right at every step and remembers the direction turned at every point. When a previously visited site is revisited, the same rotation is made as at the previous visit. Walking is continued until it terminates by revisiting the starting step. In this case the walker always has two options for the next step, except when it revisits a site, in which case it has only one option. Grassberger (1986) used this method to estimate the fractal dimension of the percolatuion hull, and obtained $D_h = 1.750 \pm 0.002$.

A lattice trail is a random walk in which no bond of the lattice is visited twice (Malakis 1975). It has been seen that these walks belong to the universality class of ordinary SAWs (Malakis 1975, Guttmann and Osborn 1988, Guttmann and Manna 1989). In kinetic growth trails the walker first determines the unvisited bonds emanating from its present position, and then chooses one of them randomly. This walk has been studied under a different name as the growing self-avoiding trail and was seen to belong to a different universality class from ordinary SAWs or lattice trails (Lyklema 1985).

Consider now an L lattice as shown in figure 3(*b*). It is a square lattice in which every bond on a path must be at right angles to its predecessor. KGTs on this lattice will satisfy the following properties: they will turn left or right at every step, visit a site at most twice and stop when there is no option for the next step (this happens only at the origin). These are exactly the same properties as the hull of a bond percolation cluster has on square lattice at the percolation threshold as described above. Therefore kinetic growth trails on the L lattice should obey the same statistics as that of the bond percolation hulls on the square lattice.

To establish this equivalence numerically we enumerated exactly ordinary trails and kinetic growth trails on the L lattice up to 37 steps (see table 3). Malakis (1976) pointed out that there is a one-to-one mapping from the set of N step trails on the L lattice on to the set of $(N-1)$-step SAWs on the Manhattan lattice. We find that C_N

Table 3. Series data for the sum of the squares of end-to-end distances (R_N^2) for SAWs and the mean square end-to-end distances ($\langle R_{NW}^2 \rangle$) for KGTs on the L lattice.

N	R_N^2	$\langle R_{NW}^2 \rangle$
1	1	1.000 000 0000 E+00
2	4	2.000 000 0000 E+00
3	12	3.000 000 0000 E+00
4	32	4.000 000 0000 E+00
5	78	5.000 000 0000 E+00
6	180	6.000 000 0000 E+00
7	400	7.000 000 0000 E+00
8	864	8.062 500 0000 E+00
9	1 818	9.387 096 7741 E+00
10	3 756	1.045 161 2903 E+01
11	7 644	1.155 645 1612 E+01
12	15 360	1.264 919 3548 E+01
13	30 504	1.407 438 0165 E+01
14	60 012	1.521 280 9917 E+01
15	117 108	1.634 297 5206 E+01
16	226 912	1.746 849 1735 E+01
17	436 764	1.904 016 9133 E+01
18	835 940	2.020 613 1078 E+01
19	1 591 790	2.136 931 8181 E+01
20	3 017 408	2.253 689 8784 E+01
21	5 695 448	2.416 940 8753 E+01
22	10 709 604	2.536 578 6504 E+01
23	20 068 182	2.656 096 0207 E+01
24	37 487 152	2.775 616 0222 E+01
25	69 816 450	2.946 308 8621 E+01
26	129 674 484	3.068 657 1352 E+01
27	240 246 868	3.190 810 1204 E+01
28	444 079 488	3.312 984 7255 E+01
29	819 036 168	3.488 423 1561 E+01
30	1 507 514 552	3.613 121 3039 E+01
31	2 769 437 372	3.737 580 7005 E+01
32	5 078 729 888	3.862 113 1263 E+01
33	9 297 786 596	4.042 523 7924 E+01
34	16 994 815 856	4.169 330 8319 E+01
35	31 017 310 020	4.295 936 7751 E+01
36	56 530 874 800	4.422 571 2589 E+01
37	102 891 837 550	4.606 985 6278 E+01

for trails on the L lattice is twice C_{N-1} for SAWs on the Manhattan lattice. The data for the sum of the mean square end-to-end distances (R_N^2) was analysed as above. With a critical point of 0.576 92, we find $2\nu+\gamma=2.844$, and hence the value of ν is 0.751. The generating function for the mean square end-to-end distances ($\langle R_{NW}^2 \rangle$) for kinetic growth trails weighted by the weights of the configurations was analysed similarly, biased at $x_c=1$, and we obtained the value of $\nu_{\mathrm{KGT}}=0.578$. We find the weights w_N for trails on the L lattice to be exactly equal to w_{N-1} for SAWs on the Manhattan lattice, reflecting the known mapping between those two problems (Malakis 1975).

In a Monte Carlo study of kinetic growth trails, we simulated one million configurations for walks of length $N=16$, 20, 24, 32, 40, 48, ..., 320, 384, 512. We fitted the mean square end-to-end distances $\langle R_N^2 \rangle$ to a form $N^{2\nu_{\mathrm{KGW}}}$. A log–log plot of $\langle R_N^2 \rangle$

against N gave the value of $\nu_{KGT} = 0.576$ (see figure 2). Utilising a second method, we studied kinetic growth trail loops in the same way as kinetic growth walks. The lattice size was varied as $L = 16, 20, 24, 32, 40, 48, \ldots, 160, 192, 256$ and the number of configurations were varied from 20 000 to 100 000. We fitted the average loop length $\langle P_L \rangle$ to a form $\langle P_L \rangle \sim L^{D_{KGT}}$. A log-log plot of $\langle P_L \rangle$ against L gives a value of $D_{KGT} = 1.77$ (see figure 2).

4. Conclusion

In this paper we have studied kinetic growth walks on the Manhattan lattice and kinetic growth trails on the L lattice. Because of the directional properties of these lattices, these walks are never trapped except at the origin of the lattice. Therefore these walks are expected to belong to the smart kinetic walk universality class rather than to the ordinary SAW universality class. Extensive numerical calculations by both exact enumeration and Monte Carlo methods show strong evidence that these walks indeed belong to the SKW universality class. Moreover it has been shown that the recently introduced hull percolation is equivalent to kinetic growth walks on the Manhattan lattice and the hull of bond percolation clusters on the square lattice at the percolation threshold is equivalent to kinetic growth trails on the L lattice.

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